International Conference of Recent Advances in Engineering-ICRAE2013
Murch 2013, Islamic Azad University, Khomeinishahr Branch, Iran

A Non-Linear Rheological Model for prediction of Stress-Strain Behavior of Polymers

Afsaneh Barekat
Department of Chemistry,
Mahshahr Branch, Islamic Azad
University, Mahshahr, Iran
Afsaneh.barekat@yahoo.com

Hossein Hosseini
Department of Chemical
Engineering, Abadan Branch,
Islamic Azad University,
Abadan, Iran
Pedram465@yahoo.com

Abstract
In this paper by using of a non-linear viscoelastic model relationship between stress-strain during the thermoforming process is obtained. By using of presented mathematical model it is possible to evaluate the kinetics of thermoforming process. Consequently the minimization of the accumulation may be realized only by minimizing of the general deformations accumulated in polymeric sheet during this process.

Keywords: Non-linear rheological model, stress-strain behavior.

Introduction

Wide applications of thermoforming are due to its high performance, simplicity, compactness and relatively low-cost equipment. These issues make it possible to produce complex, large-scale configurations and free form shapes of products. In thermoforming, a heated plastic sheet is stretched into a mold cavity by applying pressure and eventually direct mechanical loading are used [1-10]. Upon contacting of a sheet with the cold surface of the mold, the sheet deformation is terminated. The forming sequence induces a thickness variation in the final part. Besides wall-thickness variation, other problems facing the thermoforming industry are mainly physical instabilities during inflation – rupture of sheet and warpage exhibited in the final parts. There are many ways to stretch sheets: vacuum, air pressure and mechanical aids such as implementation of a plug. For increasing the quality of products such as narrow wall-thickness tolerance or elimination of frozen-in stresses, a combination of mechanical and vacuum or pressure forming methods may be implemented. The process initially involves the usage of mechanical pre-stretching with plug and then vacuum or pressure forming is applied.

Since quality of final products is characterized by their minor wall-thickness variation, so evaluation of this property provides estimation to physical properties of polymeric products such as strength and toughness. Many researches have been carried out to investigate the thermoforming process both analytically and numerically [10-13]. However, there is lack of literature about deformation processes in thermoforming and its effect on wall-thickness uniformity and frozen-in stresses, there is a need for further research. Present study was conducted to simulate deformation processes in thermoforming processes.

RHEOLOGICAL MODELING

As mentioned above, warpage prediction is very important due to processing constraints. It may finally cause system failure. Proper description of the problem is directly dependent on the correct selection of an appropriate rheological model [11]. Leonov developed a theoretical model in this area [12]:

$$\overline{\sigma} + p\overline{\delta} = 2\overline{c}W_1 - 2\overline{c}^{-1}W_2$$

$$\overline{e}_f = 1/\theta_0 G_0(T)\exp\{-\beta\, w^s/G_0(T)\} \;.\; [(\overline{c} - I_1\overline{\delta}/3)\, W_2^s] \qquad (1)$$

$$\frac{d\overline{c}}{dt} + \overline{\omega}\,\overline{c} - \overline{c}\,\overline{\omega} - \overline{c}\left(\overline{e} - \overline{e}_f\right) - \left(\overline{e} - \overline{e}_f\right)\overline{c} = 0$$

where:

$\overline{\sigma}$ :: stress tensor,

p :: Lagrange multiplier, determined by boundary condition,

$\overline{\delta}$ :: identity tensor,

$\overline{c}$ :: Cauchy strain tensor,

$\overline{e}_f$ :: flow strain rate tensor,

$\overline{\omega}$ :: vortex tensor,

$\overline{e}$ :: strain rate tensor,

$\theta_0(T)$ :: relaxation time,

$G_0(T)$ :: tensile modulus,

$$\overline{e} = \frac{\dot{\gamma}}{2}\cdot\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \overline{\omega} = \frac{\dot{\gamma}}{2}\cdot\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (2)$$

$$\overline{c} = \begin{pmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \overline{c}^{-1} = \begin{pmatrix} c_{22} & -c_{12} & 0 \\ -c_{12} & c_{11} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Now, with equations (2) and for pure shear process ($\frac{dc_{ij}}{dt} = 0$), the second and third equations of system equations (1) will be as follows:

$$\begin{cases} c_{11}^2 - c_{11}\cdot c_{22} + 2c_{12}^2 = 4\dot{\gamma}\cdot\theta_0(T)\cdot F(c_{12})\cdot c_{12} \\ c_{12}\cdot(c_{11} + c_{22}) = 2\dot{\gamma}\cdot\theta_0(T)\cdot F(c_{12})\cdot c_{22} \end{cases}$$

$$c_{22}^2 - c_{11}c_{22} + 2c_{12}^2 = 0$$

where

$$F(c_{12}) = \exp\left[(\beta - 7.8\sqrt{1 - c_{12}^2})\left(\frac{1}{\sqrt{1 - c_{12}^2}} - 1\right)\right]$$

.

From solving of equations system (3) we will have:

$$\dot{\gamma} = \frac{1}{\theta_0(T)\cdot F(c_{12})}\cdot\frac{c_{12}}{1 - c_{12}^2} \qquad (4)$$

But in practice there is a problem for application of Eq.(1). This problem arises due to the choice of selecting strain energy function $W = W(I_1, I_2)$. Most researchers use Mooney-Rivlin potential, but there are differences between experimental and theoretical results for prediction of stress and strain. Results of recent research show that in various kinematical deformations, the following potential can be used [11]:

$$W = 0.25 G_0 \left(I_1 + I_2 - 6 \right) \tag{5}$$

From the first equation of the Rheological model (1) with condition (5) and elastic tensors in equation (2), following equation for stress can be developed:

$$\sigma_{12} = G_0 (T) \cdot c_{12} \tag{6}$$

By using the strain energy function presented in Eq. (5), in conjunction with the model in Eq. (1), a mathematical description for the deformation process occurring in thermoforming can be developed.

Deformation Processes

Consider a polymeric sheet with radius (r_3) is heated for production of an axisymmetric article. It is deformed by movement of a plug with radius of r_p at constant velocity V_p, in direction of "z" axis.

The implemented material is assumed to be incompressible and isotropic. The deformation process is carried out under isothermal condition. The deformed sheet could be considered a thin shell, thus the hot polymer can be modeled as a membrane. Therefore, the bending resistance of the hot sheet is ignored and the material thickness is assumed to be small in comparison to dimensions of the material. Three different stretch ratios involving in deformation process are as follows:

$$\lambda_1 = \frac{d\xi}{d\xi_0} ; \lambda_2 = \frac{r}{r_0} ; \lambda_3 = \frac{h}{h_0} \tag{7}$$

where λ_1, λ_2 and λ_3 are the principal stretch ratios in the meridional, radial and thickness directions of the membrane, respectively. They are related together by the incompressibility condition $\lambda_1 \lambda_2 \lambda_3 = 1$ and ξ, ξ_0 are length of meridian in deformed and strainless sheet.

r, r_0 :: radii of deformed and strainless sheet, respectively.

h, h_0 :: thickness of the sheet after and before deformation, respectively.

Mechanical pre-stretching is a planar stretching (pure shear). Therefore, the following conditions exist:

$$\lambda_2 = 1 ; \qquad \sigma_3 = 0 ; \qquad \lambda_3 = \lambda_1^{-1} \tag{8}$$

With respect to the conditions and the tensors in equation (1), the following expression can be written:

$$\bar{e} = \dot{\varepsilon} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} ; \bar{\varpi} = 0 ;$$

$$\bar{c} = \begin{pmatrix} c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c^{-1} \end{pmatrix} ;$$

$$\bar{c}^{-1} = \begin{pmatrix} c^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c \end{pmatrix} \tag{9}$$

where $\dot{\varepsilon}$ :: rate of deformation in longitudinal direction.

The primary and secondary invariants of tensor $\bar{C}$ are resulted from equation (9) as:

$$I_1 = I_2 = c + 1 + c^{-1} \tag{10}$$

By utilizing equations (9) and (10), following form of equation (1) can be developed.

$$\bar{\sigma} + p\bar{\delta} = 0{,}5\, G_0 (T) \cdot \begin{pmatrix} c - c^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c^{-1} - c \end{pmatrix} \tag{11}$$

$$\bar{e}_f = \frac{1}{4\theta_0 (T)} exp\left[-\beta \left(c + c^{-1} - 2 \right) \right] \cdot \begin{pmatrix} c - c^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c^{-1} - c \end{pmatrix} \tag{12}$$

Parameter p is resulted from condition (8): $\sigma_3 = 0$. By substituting expressions (9) and (12) into equation (1):

$$\frac{dc}{dt} = 2 \left[\dot{\varepsilon} \cdot c - \left(c^2 - 1 \right) \frac{1}{4\theta_0 (T)} exp\left[-\beta \left(c + c^{-1} - 2 \right) \right] \right] \tag{13}$$

This differential equation defines kinetics of elastic strain during the deformation process of viscoelastic media. The deformation rate is defined as follows:

$$\dot{\varepsilon} = \frac{d\varepsilon^H}{dt} = \frac{d \ln \lambda}{dt} = \frac{1}{\lambda} \frac{d\lambda}{dt} \tag{14}$$

where ε^H is Hencky strain.

There are two different strains:

1) Total strain (viscous and elastic deformations) $\varepsilon^H = \frac{1}{2} ln\left[\left(\frac{\tilde{t}}{a}\right)^2 + 1\right]$ (15)

2) Elastic strain ($\varepsilon^H{}_e$): $c \equiv \lambda_e^2 = exp\left(2\varepsilon_e^H\right)$ (16)

where $a \equiv \tilde{r}_1{}^2 \frac{V_3}{V_p\theta_0(T)} \cdot \frac{1-\tilde{r}_p}{\tilde{r}_p}$; $\tilde{t} \equiv \frac{t}{\theta_0(T)}$.

Finally, from equation 11 and condition shown by equation 4 following relationship can be derived.

$$\sigma_1 \equiv \sigma = G_0(T) \cdot \left(c - c^{-1}\right) \quad (17)$$

RESULTS AND DISCUSSION

Variation of stress versus shear rate can be seen in figure 1 for ABS-sheet at temperature T=140^0 C. There is a good agreement between theoretical results from equations (4 -6) and experimental results (points).

Figure 2 shows kinetics of development of total and elastic strains. As shown there is a good agreement between theoretical (Eqs. 15 and 16) and experimental data for ABS at T=140^0 C.

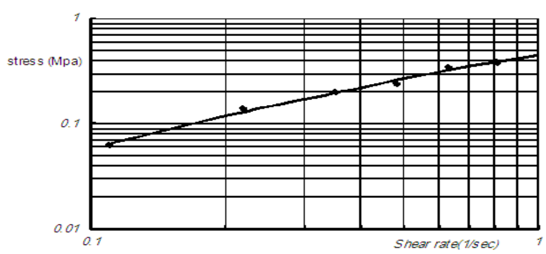

Figure 1. variation of stress vs. shear rate for ABS at T=140^0 C, β =3.55, G_0 = 0.635 MPa ; θ_0 = 0.3 sec.( ): equation 4; points: experimental data.

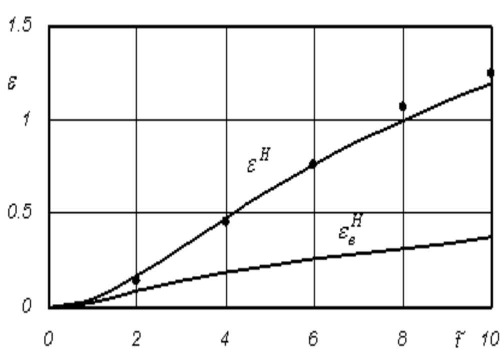

Figure 2. Kinetics of development of total and elastic strains for ABS sheet. ( ) theoretical model ; points: experimental data. β =3.55 ; θ_0 = 0.3 sec; a =3.16.

The following calculation method enables us to have a quantitative evaluation of product quality even at designing step. Followings are required data in order to perform this method:

a) Geometric parameters of product such as radius of sheet and depth of the article.
b) Thickness of the used polymer sheet.
c) Obtained results out of thermoforming process modeling, according to the description of deformation processes mentioned at the beginning of this paper.
d) Relaxation time and flexibility parameter of macromolecular chains.

By the use of above-mentioned information, it is possible to find following results:

1) It is possible to specify accumulated stresses in meridional and radial directions at plug-assisted stage and by results of deformation process modeling.
2) To calculate total and elastic strains for preventing from sheet fracture during thermoforming process.
3) To find variation of stress versus shear rate at any moment of thermoforming process application of polymeric articles.

By using of presented mathematical model in this paper it is possible to evaluate the kinetics of thermoforming process when necessary. It is important when we want to stop the deformation process at special points of mold level at vacuum forming stage. For solving this problem, it is necessary to specify only the mutual relation between boundary contact points of polymer sheet, mold surface and deformation process time. There is the following equation for vacuum thermoforming time:

$$t_{proc} = \frac{V_{arti} - V_{zag}}{\mu' \cdot s\sqrt{RT\frac{2k}{k+1}}}\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \quad (18)$$

where:
t_{proc} :: vacuum thermoforming process time,
V_{arti} :: volume of polymer article,
V_{zag} :: volume of polymer sheet after plug-assist,
K :: adiabatic coefficient of exit gas from mold hole,
T :: gas temperature,
R :: gases constant,
S :: total cross section levels of current holes on mold,
μ'::dimensionless coefficient which is a sign of gas consumption in thermoforming equipments (0.4 – 0.6).

Now, it is possible to have a suitable relation for kinetics of thermoforming process by equation of 22.

- It is well known that the stage of vacuum-forming with a pre-stretched sheet occurs too quickly for experiencing of relaxation processes in the polymeric sheet. This will result in this fact that almost all accumulated deformations in polymeric sheet within this stage are elastic. Consequently, the minimization of the accumulation may be realized only by minimizing of the general deformations accumulated in polymeric sheet during this stage. For practical purposes, this means that the profile of the pre-stretched sheet should be maximally approximated by the profile of the final product which could be assured based on the application of a plug with the respective radius.

Conclusion

It is well known that the stage of vacuum-forming with a pre-stretched sheet occurs too quickly for experiencing of relaxation processes in the polymeric sheet. This will result in this fact that almost all accumulated deformations in polymeric sheet within this stage are elastic. In contrast to the second stage, the stage of plug-assisted can be regulated in the sense that it is technically possible to control the motion of the plug. This creates a practical opportunity at this stage to organize the relaxation process of elastic deformations accumulated in a polymeric sheet during the process of plug-assist forming.

References

[1] Martin P.J., Duncan P.: The Role of Plug Design in Determining Wall Thickness Distribution in Thermoforming, Polymer Engineering and Science, **47**, 804-813 (2007).

[2] Scott C.E.,Virkler T.L.: Effect of ABS Rubber particles on rheology, melt failure and thermoforming, Polymer Engineering and Science, **42**, 1541-1557 (2002).

[3] Erchiqui F., Imad F., Mouloudi A.: Caractérisation viscoélastique du comportement d'une membrane thermoplastique et modélisation numérique de thermoformage, Canadian Journal of Chemical Engineering, ***88***, 116-125 (2010).

[4] Throne, J.L.: *Thermoforming*, Hanser publishers, Munich (1987).

[5] Lim T.: A three-level hierarchical approach in modeling sheet thermoforming of knitted-fabric composites, International Journal of Mechanical Science, 45, 1097-1117 (2003).

[6] Allard R., Charrier J.M., Ghosh A., Marangou M., Wu R.: An Engineering Study of the Thermoforming Process: Experimental and Theoretical Considerations, Journal of Polymer Engineering, 6, 363-394 (2012).

[7] Sala G., Landro L., Cassago D.: A numerical and experimental approach to optimise sheet stamping technologies: polymers thermoforming, Materials & Design, **23**, 21-39 (2002).

[8] Aroujalian A., Ngadi M.O, Emond J.P.: Wall thickness distribution in plug- assist vacuum formed strawberry containers, Polymer Engineering and Science, **37,** 178-182 (1997).

[9] Hosseini H., Berdyshev B.V., Mehrabani A.: Rheological Modeling of plug-assist thermoforming, Journal of Applied Polymer Science, **101**, 4148-4152 (2006).

[10] Hosseini H., Berdyshev B.V., Mehrabani A.: A solution for warpage in polymeric products by plug-assisted thermoforming, European Polymer Journal, **42,** 1836-1841 (2006).

[11] Oden J.T.: Finite Elements of Non-Linear Continua, McGraw-Hill, New York (1972).

[12] Rosenzweig N., Narkis M.: Sintering Rheology of amorphous polymers, Polymer Engineering and Science, **21**, 1167-1170 (1981).

[13] Ghafur M.O., Koziey B., Vlachopoulos J.: Simulation of thermoforming and blowmolding-theory and experiments, Kluwer Academic Publishers, New York (1995).